

DYNAMICS OF A MATHEMATICAL MODEL OF PLANTHOPPER-BEETLE INTERACTIONS FOR CONSERVATION IN RICE FIELD ECOSYSTEMS USING HOLLING TYPE II AND CROWLEY-MARTIN FUNCTIONAL RESPONSES

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Article Info	ABSTRACT
Article history: Received February 17th, 2026 Revised February 21th, 2026 Accepted March 24th, 2026 *Corresponding Email: diansavitri@unesa.ac.id	Advances in science and technology have led to new ways of understanding how living things interact within ecosystems. One such method is the creation of mathematical models to explain the relationships and changes in population numbers in numerical terms. This study aims to create a model of the interaction between planthoppers, which are the main pests of rice plants, and beetles, which are natural predators of planthoppers. This model uses two types of response functions, namely Holling type II and Crowley-Martin, which describe how predators' prey while considering the effects of satiation and interference between other predators. The analysis was conducted analytically and numerically to determine the equilibrium point, local stability, and the impact of parameter changes on population changes. The results show that the system has a stable positive equilibrium point under certain conditions. Variations in certain parameters can cause changes in population stability within the system. Models with two response functions (Holling Type II and Crowley-Martin) provide more realistic results than single models. This research contributes to the development of mathematical ecology models that provide a deeper understanding of predator-prey interactions and can serve as a basis for more effective pest population control in agricultural environments. Keyword: <i>Mathematical Models, Prey-Predator, Population Dynamics, Conservation, Ecosystem</i>

Introduction

Mathematical modeling to describe interactions between populations by considering how populations behave in overcoming problems. Pests are animals that harm plants, causing suboptimal plant growth and development. Until now, pests have generally been controlled by using chemical pesticides. However, the use of these pesticides can have negative effects, such as side effects on the pests themselves and the ecosystem around the plants. Due to these adverse effects, the use of chemical pesticides is gradually being reduced. One alternative used by farmers is biological pest control. Biological control is carried out by using living organisms that are natural enemies of pests, so that the number of pests can be reduced to a certain level, thereby minimizing losses (Lehman, 2010). Natural enemies play an important role in reducing pest populations to the lowest possible level. Various types of natural enemies of pests include parasitoids, predators, and insect pathogens (Surtikanti, 2020). By involving predators as natural enemies, it is hoped that pest growth can be slowed down and controlled so that it does not disturb the balance of the ecosystem (Srinivasu and Prasad, 2011). The phenomenon of predators preying on pests can be modeled mathematically by considering the predation rate

and prey behavior, such as the prey's fear of predators and anti-predator behavior, as explained by Afidah dan Savitri (2021) and Abadi and Savitri (2015).

Biological control has begun to be implemented as a more environmentally friendly alternative. One predator that plays an important role in the rice field ecosystem is the predatory beetle (*Paederus fuscipes*), which preys on stink bugs (*Nilaparvata lugens*), the main pest of rice plants. The presence of predatory beetles is expected to slow down the growth of the stink bug population and maintain the stability of the agricultural ecosystem. However, the rate of predation on pests is not constant. Predation activity can be influenced by many factors such as handling time, predator saturation, and interference between predators. To describe this phenomenon mathematically, a predator-prey model was developed based on the interaction between two populations, namely prey (planthoppers) and predators (beetles).

According to Xiao and Ruan (2001), the classic Lotka–Volterra model describes the interaction between two populations, prey and predator, without considering the predator's limited capacity to prey. This model assumes that the rate of predation is directly proportional to the number of prey available. However, predators have limited time and energy to handle each prey, so a more realistic modification of the model is needed. One development of this model is the use of the Holling type II response function, which takes into account the handling time for each predation process. This function explains that the predation rate increases with the number of prey until it reaches a saturation point when the predator can no longer increase its feeding activity. This approach is suitable for describing the feeding behavior of predatory beetles (*Paederus fuscipes*) on brown planthoppers (*Nilaparvata lugens*), because beetles need a certain amount of time to process their prey. According to Panja (2021), the Crowley–Martin response function was developed to account for predator interference, a condition in which predation effectiveness decreases due to negative interactions between predator individuals. This model provides a more complex picture than the Holling type II model because it can show phenomena such as stability, periodic oscillations, and the possibility of bifurcation.

Several follow-up studies show that the stability of the predator–prey system can be improved by considering environmental factors or providing additional food for predators. Therefore, combining the two Holling type II and Crowley–Martin response functions provide a more comprehensive approach to analyzing the dynamics of interactions between planthopper populations and predatory beetles in rice field ecosystems. This study aims to construct a mathematical model of the interaction between brown planthoppers as the main pest and predatory beetles as natural enemies using these two response functions. In addition, analytical and numerical stability analyses of the system are performed to understand the effect of parameter changes on population dynamics. The results of this study are expected to provide a scientific basis for the development of sustainable ecology-based pest control strategies.

Methods

This section of the research method systematically describes the steps taken in the process of designing, analyzing, and interpreting the developed mathematical model (Alebraheem, 2012).

Stage I: Preliminary Study and Literature Review

The initial stage of the research began with a comprehensive literature review of prey-predator models and their various developments in the context of dynamic systems. The focus of the study was directed at understanding the basic characteristics of prey-predator relationships, including the concepts of population growth, predation interactions, and the adaptive behavior of the two interacting entities. In addition, references related to assumptions and general parameters commonly used in prey-predator modeling were collected. The results of this literature review were used as a basis for formulating the initial model structure and establishing assumptions that are theoretically and applicationally relevant in a broader scientific context.

Stage II: Construction of a Mathematical Prey-Predator Model

This stage aims to construct a prey-predator mathematical model based on a system of differential equations, referring to the concepts studied in the previous stage. This model represents two populations, namely prey as organisms that develop naturally but can be targeted for attack, and predators as organisms that survive by preying on prey. In the construction process, important elements are included, such as the growth rate of prey, predation effectiveness, prey defense behavior, and predator dependence on prey. The resulting model is expected to be able to describe the dynamics of population interactions in mathematical form, so that it can be analyzed quantitatively using dynamic and numerical approaches.

Stage III: Dynamic Analysis, Bifurcation, and Numerical Simulation

This stage is the core of the research, which focuses on analyzing the behavior of the constructed prey-predator model. The purpose of this stage is to evaluate the stability of the system and study population dynamics under various conditions through mathematical approaches and computer simulations. The details of this analysis are as follows:

1. **Determination of the Equilibrium Point**
The equilibrium point of the system is obtained by finding the condition where all model variables remain unchanged over time. This point reflects the stationary state of the prey and predator populations, which serves as a reference for further analysis.
2. **Determining the Conditions for the Existence of the Model's Equilibrium Point**
A search was conducted for mathematical conditions that ensure that the equilibrium point exists and is biologically meaningful (for example, has a positive value), in accordance with the parameter values used.
3. **Local Stability Analysis of Equilibrium Solutions Around Equilibrium Points**
A local analysis is performed to determine whether the equilibrium point is stable or unstable when a small disturbance occurs in the system. The evaluation is carried out by analyzing the eigenvalues of the Jacobian matrix formed from the system around the equilibrium point. The stability criteria are determined using the Routh-Hurwitz approach or directly from the sign of the eigenvalues.
4. **Numerical Simulation and Numerical Continuation**
Numerical simulations were applied to observe the behavior of the system over a certain period of time, using numerical methods such as Runge-Kutta implemented in Python software. The simulation results were visualized in the form of graphs and phase portraits to facilitate interpretation of the system dynamics. Numerical continuation was performed by varying the system parameters to observe the emergence of changes in stability properties.

Stage IV: Interpretation of Results and Scientific Implications

The final stage aims to summarize and interpret the results of the entire prey-predator model analysis and simulation process. The data obtained is reanalyzed to understand the patterns of change in prey and predator populations, as well as how certain parameters can affect system dynamics. Interpretation is carried out in the context of scientific development, particularly in the study of complex systems and the interaction of two entities within the framework of science and technology. The results of this study are expected to contribute to the mathematical modeling of dynamic systems and serve as a reference for further research in relevant fields, including computational biology, artificial ecological systems, and simulation-based population control.

After the methodological stages have been thoroughly designed, the next step is to construct a mathematical model that represents the dynamics of biological interactions between the two species under study. In this section, the model formulation is developed by considering biological aspects, basic assumptions, and the most appropriate response function to describe the predation mechanism in rice field ecosystems. At this stage, the mathematical approach focuses on developing a model structure that can represent population dynamics more accurately by modifying existing classical models.

Simple interaction model of two populations

Interactions between prey and predators represent predator consumption of prey, with predator populations depending on prey populations as their sole food supply, and prey having limited food supplies. If there is a predator population, let us say $y(t)$ exists. This means that the growth rate of prey is given by prey growth minus the rate of predation. The natural growth rate is a combination of births and deaths, meaning that if there is no prey, there is no energy to support births. If there is a food supply (prey), the growth of the predator population is proportional to the death of the prey population. Changes in the predator population are due to prey growth minus natural deaths.

Lotka-Volterra predator-prey model

The model introduced by Lotka (1925) and Volterra (1926), assumes that the prey population grows exponentially if there are no predators. If there is a predator population, then the growth rate of the prey is given by the growth of the prey minus the predation rate. This model is known as the Lotka-Volterra model, which is expressed as follows:

$$\begin{aligned}\frac{dx}{dt} &= rx - \beta xy \\ \frac{dy}{dt} &= \rho xy - \mu y\end{aligned}\quad (1)$$

The model presents the population density of prey and predators in x and y , with parameters r , β , ρ , and μ being positive constants representing the intrinsic growth rate of the prey population.

The model used in this study is a modification of the Lotka-Volterra predator-prey model, which describes the interaction between two populations, namely the brown planthopper (*Nilaparvata lugens*) as prey ($x(t)$) and the predatory beetle (*Paederus fuscipes*) as predator ($y(t)$). This model is modified with two response functions, namely:

a. Holling type II

According to Afidah and Savitri (2021), a model that considers handling time in predators. The study of the two-species Lotka-Volterra predator-prey model was initiated by (Maynard dan Slatkin, 1973 & Hu dan Cao, 2017) with the Michaelis-Menten-Holling response function. The model assumes that the prey population grows logistically.

$$\frac{dx}{dt} = rx \cdot \left(1 - \frac{x}{k}\right) \cdot \frac{mxy}{a+x} \quad (2)$$

Based on the assumptions described in Afidah and Savitri (2021), a model of interaction between one prey and one predator was obtained using the Holling type II response function, taking into account cannibalistic behavior in predators as follows:

$$\begin{aligned}\frac{dx}{dt} &= rx \cdot \left(1 - \frac{x}{k}\right) \cdot \frac{mxy}{1+ax} \\ \frac{dy}{dt} &= \frac{nxy}{1+ax} + cy - \beta y - \frac{cy^2}{y+d}\end{aligned}\quad (3)$$

In this system, $x(t)$ represents the prey population at time t , while $y(t)$ represents the predator population at the same time. The natural growth of the prey population is described using a logistic growth model with a natural growth rate of prey, namely r . The growth of the predator population is described using the Lotka-Volterra model, considering the Holling type II response function. This system also considers cannibalism among predators in order to survive. In this model, cannibalism is assumed to reduce the predator population by injuring or even killing them.

b. Crowley-Martin

The predator-prey model that considers predator interference is introduced through the Crowley-Martin response function, which is an extension of the Holling type II response function. In this

response function, the predation rate is influenced not only by the number of prey, but also by negative interactions between predators that reduce predation effectiveness. The general model can be written as follows:

$$f(x) = \frac{mxy}{(1 + ax)(1 + cy)} \quad (4)$$

Tripathi (2020) has presented and examined a two-dimensional continuous-time dynamic system that models predator-prey interactions integrating the Crowley–Martin functional response. The Crowley–Martin functional response is similar to the Beddington–DeAngelis functional response but contains an additional term that describes mutual disturbance by predators at high prey densities. The study considered the local stability of various equilibrium points and the global asymptotic stability of positive equilibrium points to understand the dynamics of the model system.

Results and Discussion

The following model Zhang (2022), which examines the interaction between prey, predator, and super predator using the Crowley-Martin response function and Holling Type I combines protection for both predators.

$$\begin{aligned} \frac{dx}{dt} &= ry - mx - \alpha xz - d_1x \\ \frac{dy}{dt} &= mx - sy^2 - \frac{\beta yz}{(1 - \alpha y)(1 - bz)} - d_2y \\ \frac{dz}{dt} &= p\alpha xz + \frac{q\beta yz}{(1 + ay)(1 + bz)} - \delta z^2 - d_3z \end{aligned} \quad (5)$$

Based on equation (5), a model of 1 prey and 1 predator was constructed using type II and Crowley-Martin response functions.

$$\begin{aligned} \frac{dx}{dt} &= rx \cdot \left(1 - \frac{x}{k}\right) \cdot \frac{qxy}{ax + 1} \\ \frac{dy}{dt} &= -sy \cdot \frac{uxy}{bdxy + bx + dy + 1} \end{aligned} \quad (6)$$

Based on equation (5), a new model was constructed considering the limitations of the classic Lotka–Volterra model. This model only describes the interaction between prey and predator populations, if the predation rate increases linearly with the number of preys. Predators have limited time and energy to deal with each prey, so the predation rate cannot continue to increase proportionally. To describe more realistic conditions, two response functions were used, namely Holling type II and Crowley–Martin. The Holling type II response function considers the handling time in the predation process, which causes the predation rate to increase with the number of prey until it reaches a saturation point. Meanwhile, the Crowley–Martin response function adds the effect of predator interference, which is a decrease in predation effectiveness due to negative interactions among predators.

The combination of these two response functions results in a more comprehensive equation (6) because it is able to represent two important aspects in predator–prey dynamics: time constraints and the influence of interference between predators. This model also allows for various system dynamics to emerge, such as stable conditions, periodic oscillations, and changes in system behavior due to parameter variations. Overall, the construction of a model with two response functions provides a more realistic approach to analyzing the interaction between planthoppers as prey and predatory beetles as

their natural enemies. In addition, analytical and numerical analysis of system stability can provide a deeper understanding of the influence of parameters on population dynamics and serve as a basis for the development of sustainable pest control strategies.

Assumptions were then made as a basis for constructing the proposed mathematical model to demonstrate its biological relevance. The model consists of two populations with prey-predator characteristics. Next, a dynamic analysis was conducted to determine the model of interaction dynamics between populations in the rice field ecosystem. The steps taken in the analysis stage are as follows:

- (i) Determine the equilibrium point. The equilibrium point is said to exist if each element has a non-negative value.

$$E_1 = [x = 0, y = 0]$$

$$E_2 = [x = k, y = 0]$$

$$E_3 = [x = x^*, y = y^*]$$

Where x^* is the positive real solution of equation (3.2), so that we obtain

$$Ax^{*3} + Bx^{*2} + Cx^* + D = 0$$

$$A = (abdrs)^3$$

$$B = (-abdkrs + adrs + bdrs)^2$$

$$C = (-adkrs - bdkrs - bkqs + drs + kqu)$$

$$D = (-dkrs - kqs)$$

Next, the value of (y^*) that satisfies (y^*) is given by:

$$(y^*) = \frac{1}{kq} (r(a(x^*)^3) + (b(x^*)^2) + (c(x)^*) + (ak(a(x^*)^3) + (b(x^*)^2) + (c(x)^*)))$$

Local stability analysis

Analysis of the local stability of the equilibrium solution of the model around the equilibrium point. The local analysis stage determines the type of system stability around the equilibrium point by analyzing the eigenvalues of the Jacobi matrix around the equilibrium point.

$$J = \begin{bmatrix} r \cdot \left(1 - \frac{x}{k}\right) - \frac{rx}{k} - \frac{qy}{ax+1} + \frac{0.5xy}{(2.4+x)^2} - \frac{qxya}{(ax+1)^2} & \frac{-qx}{ax+1} \\ \frac{uy}{bdxy+bx+dy+1} - \frac{uxy(bdy+b)}{(bdxy+bx+dy+1)^2} & -s + \frac{uy}{bdxy+bx+dy+1} - \frac{uxy+(bdx+d)}{(bdxy+bx+dy+1)^2} \end{bmatrix}$$

- a. Stability of equilibrium points $E_0 = (0,0)$.

$$J1 = \begin{bmatrix} r & 0 \\ 0 & -s \end{bmatrix}$$

The values are $\lambda_1 = r$ and $\lambda_2 = -s$. Since all parameters are positive, λ_2 is negative and λ_1 is positive. Therefore, E_1 is not a saddle point.

- b. Stability of equilibrium points $E_1 = (k, 0)$

$$J2 = \begin{bmatrix} -r & \frac{-qk}{ak+1} \\ 0 & -s + \frac{uk}{bk+1} \end{bmatrix}$$

The values are $\lambda_1 = -r$ and $\lambda_2 = -\frac{bks-uk+s}{bk+1}$. Since all parameters are positive, dengan λ_1 is negative and λ_2 is negative on condition that $bks + s > ku$, Therefore, E_1 stable.

c. Stability of equilibrium points $E_3 = (x^*, y^*)$

$$J_3 = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

The equilibrium point $E_3 = (x^*, y^*)$ is stable if $(A_{11} + A_{22}) < 0$ and $(-A_{11} - A_{22})^2 - 4(A_{11}A_{22} - A_{12}A_{21}) < 0$, The third interior equilibrium point of the coexisting populations requires further analysis and is only discussed numerically.

The characteristic equation obtained at the E_3 equilibrium point is a third-degree polynomial, so to determine its stability, it is sufficient to check the sign of the coefficients. However, the stability analysis of E_3 cannot always be easily solved analytically because it depends on the model parameters. Therefore, numerical simulations were performed to investigate the behavior of the system and ensure the asymptotic stability of the equilibrium point E_3 under certain conditions using a numerical approach based on the Routh-Hurwitz principle.

(ii) Numerical Simulation Stage

The simulation was conducted to illustrate the results of the analysis that had been carried out. To facilitate the solution using the fourth-order Runge-Kutta method, Python software was used. The display in the numerical simulation took the form of a graph showing the dynamics of changes in both populations and a phase portrait of the system solution. In this simulation, only cases 1, 2, and 3 were performed, namely numerical simulations that show the stability of equilibrium points $E_1 = [x = 0, y = 0]$, $E_2 = [x = k, y = 0]$, and $E_3 = [x = x^*, y = y^*]$. The numerical simulation shows the conditions listed in the local stability analysis stage. The numerical results are displayed to illustrate the stability of equilibrium points E_1, E_2 , and E_3 . Then, parameter values are given based on assumptions and hypotheses.

Table 1. Table of Parameters and Explanations

Parameter	Description	Value
x	Planthopper population	-
t	Beetle population	-
r	Prey growth rate	8.55
K	Environmental carrying capacity	6.0
q	The rate of predation when interacting with prey	3.0
u	Constant efficiency of conversion of predation results in predator growth	2.4
a	The time required to prey on prey	1.2
b	Boredom due to prey density, Predators busy because of abundant prey)	1.3
d	Predator fatigue during hunting	0.9
s	Natural death rate of predators	0.4

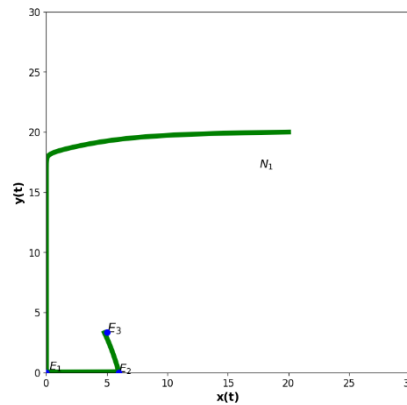


Figure 1. Portrait of the system phase (3.3) when parameter $u = 2.4$.

The visualization shows the dynamics of prey–predator interactions at those parameter values and the movement patterns of phase trajectories toward the relevant equilibrium points.

Table 2. Table of eigenvalues and stability when parameters are based on Table.1

Equilibrium Point	Eigen value	Stability Properties
(0,0)	-0.4000000000000000, 8.550000000000000	unstable
(6,0)	-8.550000000000000, 1.236363636000000	unstable
(6.041696322, 0.1633981674)	-5.88233327319604, -0.313652992503964	stable

There are three existing equilibrium points:

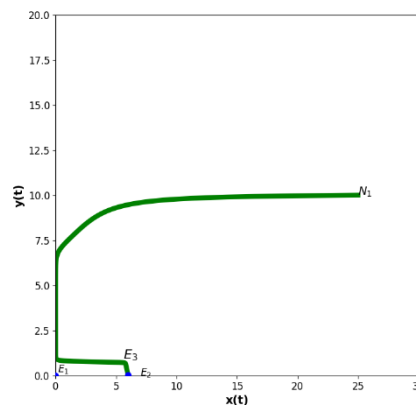
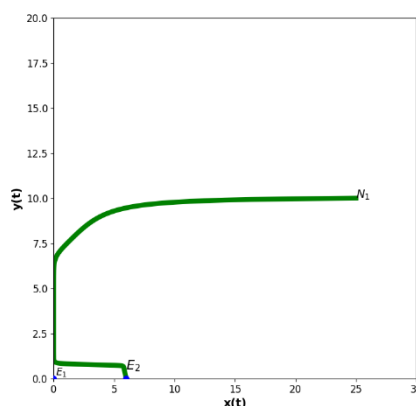


Figure 2. Portrait of the system phase (3.3) when stable at E_3

Stability analysis of the equilibrium point for parameter value $u = 0.6$ with other parameters according to Table 1. Under these conditions, the stability condition $b k s + s > u k$ is satisfied, namely $3.52 < 14.4$, resulting in $3.52 < 3.6$ which indicates that equilibrium point E_2 is unstable, while equilibrium E_3 is stable and exists. This result indicates that the system dynamics tend toward E_3 when $u = 0.6$, while E_2 cannot be maintained as a long-term equilibrium state.

Table 3. Table of eigenvalues and stability when parameters are based on Table.1

Equilibrium Point	Eigen value	Stability Properties
(0,0)	-0.4000000000000000, 8.550000000000000	unstable
(6,0)	-8.550000000000000, 0.00909090910000000	unstable
(5.993546281, 0.02511349551)	-8.53268040233311, -0.00889012075489485	stable

**Figure 3.** Portrait of the system phase (3.3) when stable at E_2

Analysis of system stability when parameter $u = 0.5$ (constant efficiency of conversion of predation results into predator growth) with other parameters according to Table 1. Under these conditions, the condition $bks + s > uk$ remains satisfied, namely $3.52 < 14.4$. Furthermore, we obtain $3.52 > 3$, which indicates that equilibrium point E_2 is **stable**, while equilibrium point E_3 **does not exist**, consistent with the results in **Table 4**.

Table 4. Table of eigenvalues and stability when parameters are based on Table.1

Equilibrium Point	Eigen value	Stability Properties
(0,0)	-0.4000000000000000, 8.550000000000000	unstable
(6,0)	-8.550000000000000, -0.0590909091000000	stable

The dynamics of both prey and predator populations can be seen in the following *time series* graph based on the parameters listed in **Table 1**. Time series graph of the system (3.3) when the parameter u is equal to 2.4. The trajectory shows that the system solution moves towards a stable equilibrium point E_3 with equilibrium values $x = 4.997098345$ and $y = 3.332989260$. Time series graph of the system (3.3) when the parameter u is equal to 0.6. The system solution returns to a different stable equilibrium point E_3 , namely at $x = 5.993546821$ and $y = 0.02511349551$. Both graphs show that changes in the value of the uuu predation conversion efficiency parameter affect the position of the equilibrium point, but the system still produces a stable equilibrium for values of $u = 2.4$ and $u = 0.6$.

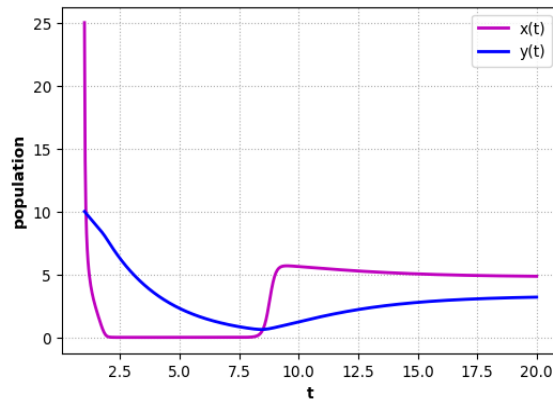


Figure 4. Time series graph of the system (3.3) when the parameter u is equal to 2.4

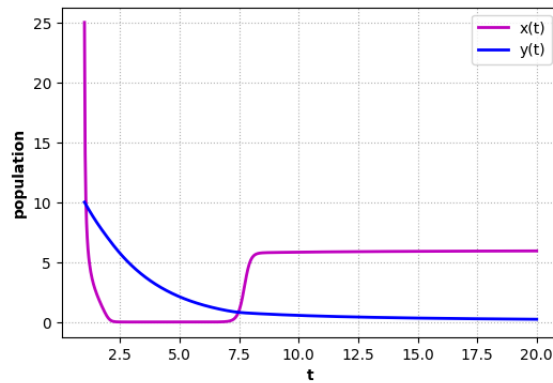


Figure 5. Time series graph of the system (3.3) when the parameter u is equal to 0.6

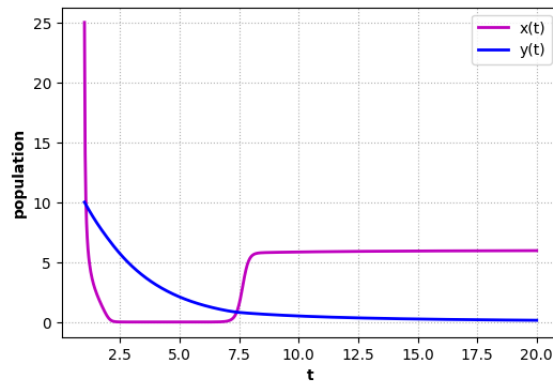


Figure 6. Time series graph of the system (3.3) when the parameter u is equal to 0.5

At this parameter value, the system moves toward the stable equilibrium point E_2 with equilibrium values $x = 6.041696322$ and $y = -0.01633981674$, while equilibrium point E_3 does not exist. These results are consistent with the stability analysis in **Tabel 4**, where a decrease in the value of u causes the loss of equilibrium E_3 (Kar et al., 2012).

Conclusion

The results of this study differ from previous models because they do not consider competition between prey, and change the method of predation by using two response functions simultaneously, namely Holling type II and Crowley–Martin. These two functions provide a more realistic picture of the time required for prey to be handled and the disturbances that occur between predators during the predation process, so that the dynamics formed are more in line with the biological conditions of planthoppers and predatory beetles in rice field ecosystems. The model produced several equilibrium points that are meaningful in a biological context. Three stable equilibrium points were produced, namely E_1 , E_2 (by changing the parameter u to 0.5), and E_3 (by changing the parameter u to 0.6). Numerical simulations were performed by changing the parameter values, and three stable equilibrium points were obtained, namely E_1 , E_2 , E_3 under certain conditions. The simulation results show that the parameter of the conversion efficiency constant from predation to predator growth affects changes in the system solution. This finding provides an understanding of the conditions that enable stable coexistence between pests and their natural enemies and can be used as a scientific basis for designing more effective, ecological, and sustainable pest control strategies.

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