CONNECTIVITY INDICES OF POWER GRAPHS OVER DIHEDRAL GROUPS OF A CERTAIN ORDER

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ABSTRACT

The dihedral group is a mathematical structure generated by rotational and reflection symmetries. In this study, the representation of the group is described using a power graph, where all elements of the group are treated as vertices, and two distinct elements are considered adjacent when one is a power of the other. By analyzing the structural patterns of the resulting power graphs, various connectivity indices can be determined, particularly for dihedral groups whose orders are powers of a prime number. This research focuses on six specific connectivity indices: the first Zagreb index, the second Zagreb index, the Wiener index, the hyper-Wiener index, the Harary index, and the Szeged index. The significance of this study lies in showing how algebraic structures can be translated into graph-theoretical frameworks, providing deeper insights into both mathematical theory and potential applications in network science.

Keyword: Connectivity Index ; Power Graph ;Dihedral Group.

Introduction

One way to understand groups is by representing them using graphs. This line of research was first introduced by Arthur Cayley in 1843, and the resulting graphs are now known as Cayley graphs. Cayley defined the representation of a group G on a graph by viewing all elements in the group as vertices, and an element is connected if there is a $x, y \in G$ is connected if there exists $a \in A \subset G$ such that $xa = y$. Research progresses by changing the group object to a subgroup as in (Luo et al., 2011) and (Assari & Hosseinzadeh, 2013). Further research on group representation in graphs is then carried out by changing the definition of adjacency such as the power graph. (A. V. Kelarev & Quinn, 2000), element oder graph (Munandar, 2022a) and coprime graph (Ma et al., 2014).

The concept of representing groups through power graphs of a finite group G (denoted by $\Gamma(G)$ was first introduced by in (A. V. Kelarev & Quinn, 2000). In this definition, the elements of G are represented as vertices, and two elements $a, b \in G$ are adjacent if there exists a natural number k such that $a^k = b$. Under this definition, the resulting graph is directed since a being adjacent to b does not necessarily imply that b is adjacent to a. The research was then carried out on the structure of semigroups (A. V. Kelarev & Quinn, 2002) and developed in (A. V. Kelarev & Quinn, 2004) by looking at the combinatoric properties that arise from the graph.

Motivated by the work of Kelarev and Quinn, Chakrabarty et al. (Chakrabarty et al., 2009) developed power graphs on semigroups and cyclic groups where the graphs are undirected. The graph is undirected, because Chakrabarty defines the element connectivity in the power graph as follows: for any $a, b \in G$, vertex a adjacent to b if and only if $a \neq b$ and $a^k = b$ or $a = b^k$ for a natural number k. Some researches that develop power graph version of undirected graph on finite groups can be found in the following studies (Cameron, 2010), (Cameron & Ghosh, 2011) and (Ali et al., 2022). While the discussion of the power graph on the torsion-free group was presented in (Cameron et al., 2019). Research from (Cameron & Jafari, 2020) discusses the connectivity of power graphs on certain groups.

On the other hand, graphs are known as a theory that can be applied in many fields. One of them is in the field of chemistry which is related to the connectivity index. Connectivity index can be used as a descriptor of chemical molecules that are calculated based on the graph of the chemical compound formed. There are several types of connectivity indices that are interesting to discuss including hyper-Wiener, Harary, first Zagreb, second Zagreb, and Szeged. Some of these indices can be used to analyze the chemical structure formed from the graph. (N.Trianjstic, 1987), and can also be used to analyze the chemical properties of paraffin (Wiener, 1947). The discussion about the connectivity index of the group on the graph is done in (Zahidah et al., 2021)In this research, the connectivity index of the coprime graph on the dihedral group for $n = 2^k$ and $n = p$ with p are prime numbers. This research will discuss the connectivity index of the power graph over the dihedral group for order $n = p^k$ with p prime numbers. The importance of this study lies in bridging group theory and graph theory through connectivity indices, providing not only theoretical contributions to algebra and discrete mathematics but also potential applications in chemistry, network analysis, and related interdisciplinary fields.

Methods

The study was conducted by looking at the patterns that appear on the power graph over the dihedral group for order $n = p^k$ with p prime numbers. The discussion starts by looking at some things related to graphs and basic results about dihedral groups. Terminology related to graphs is taken from (Munandar, 2022b), and for the sake of the integrity of this article, some definitions that are needed in the following discussion are given below:

Definition 2.1 (J. Gallian, 2021). *The dihedral group denoted D_{2n} is*

$$D_{2n} =< a, b | a^n = b^2 = e, b^{-1}ab = a^{-1} >.$$

Base on (Febriantono et al., 2024), the order of each element in the dihedral group is as follows

$$o(a^i b^j) = \begin{cases} \dfrac{n}{gcd(i, 2n)}, & j = 0 \\ 2 \quad , & j = 1 \end{cases}$$

This study investigates the connectivity indices of coprime graphs over generalized quaternion groups. The indices considered in this research include the First Zagreb Index, Second Zagreb Index, Wiener Index, Hyper-Wiener Index, Harary Index, and Szeged Index. The definitions of these six indices are presented as follows.

Definition 2.2 (Das et al., 2015) *Given a simple connected graph Γ. The First Zagreb index over Γ denoted by $M_1(\Gamma)$ is defined as follows*

$$M_1(\Gamma) = \sum_{v \in V(\Gamma)} \deg(v)^2$$

with $deg(v)$ is the vertex degree v which is the number of edges incident to v.

Definition 2.3 (Das et al., 2015) *Given a simple connected graph Γ. The Second Zagreb index over Γ denoted by $M_2(\Gamma)$ is defined as follows*

$$M_2(\Gamma) = \sum_{uv \in E(\Gamma)} deg(u)\, deg(v)$$

where $deg(v)$ is the vertex degree v.

Definition 2.4 (Dobrynin et al., 2001) *Given a simple connected graph Γ. The Wiener index over Γ denoted by $W(\Gamma)$ is defined as follows*

$$W(\Gamma) = \sum_{u,v \in V(\Gamma)} d(u, v)$$

with $d(u, v)$ is the distance between vertices u and v is the number of edges in the shortest path connecting u and v.

Definition 2.5 (Yu et al., 2019) *Given a simple connected graph Γ. The hyper-Wiener index over Γ denoted by $WW(\Gamma)$ is defined as follows*

$$WW(\Gamma) = \frac{1}{2}\left(W(\Gamma) + \sum_{u,v \in V(\Gamma)} d(u, v)^2\right)$$

where $d(u, v)$ is the distance between vertices u and v.

Definition 2.6 (Xu & Das, 2011) *Given a simple connected graph Γ. The Harary index over Γ denoted by $H(\Gamma)$ is defined as follows*

$$H(\Gamma) = \sum_{u,v \in V(\Gamma)} \frac{1}{d(u, v)}$$

where $d(u, v)$ is the distance between vertices u and v.

Definition 2.7 (Das & Gutman, 2009) *Given a simple connected graph Γ and e is an edge in Γ. The Szeged index in Γ denoted by $Sz(\Gamma)$ is defined as follows*

$$Sz(\Gamma) = \sum_{e \in E(\Gamma)} |N_1(e|\Gamma)||N_2(e|\Gamma)|,$$

where $N_1(e|\Gamma) = \{w \in V(\Gamma) | d(w, u) < d(w, v)\}$ and $N_2(e|\Gamma) = \{w \in V(\Gamma) | d(w, v) < d(w, u)\}$.

Results and Discussion

The discussion is organized into two parts: the first part examines the patterns that emerge in the power graph of the dihedral group D_{2n} for $n = p^k$ with p being a prime number, while the second part analyzes the connectivity indices of the graph.

1.1 Power Graph over Dihedral Group

The first result to be presented in this article is the power graph formed over the dihedral group. Before discussing this further, the definition of power graph is first given.

Definition 3.1 (Chakrabarty et al., 2009) *Suppose G is a group and $a, b \in G$. The power graph of G (denoted $\Gamma(G)$) is a graph consisting of the set of vertices which are elements in the group, and the vertices are $a, b \in G$ are mutually adjecent if and only if there exists a natural number n such that $a^n = b$ or $a = b^n$.*

We start our discussion by looking at the patterns that emerge in power graphs over dihedral groups. The following example can illustrate the power graph formed over dihedral group.

Example 3.2. Given a dihedral group $D_8 = \{e, a, a^2, a^3, b, ab, a^2b, a^3b\}$. The power graph of the group is as follows

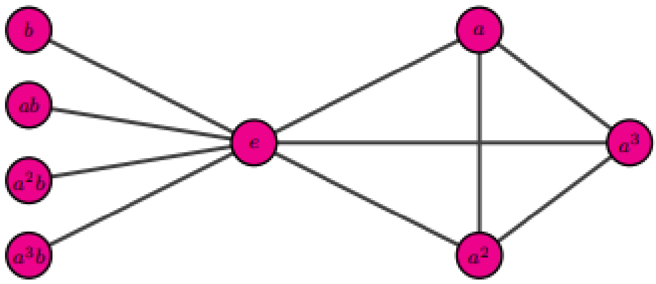

Figure 1. Power graph of the group D_8

As the example of power graph above, in the case of $n = 4 = 2^2$, the power graph formed consists of 2 mutually disjoined subgraphs. If the set of elements $S = \{e, b, ab, a^2b, a^3b\}$ then a subgraph in the form of a star graph is formed, while if the set of elements is chosen $\{a, a^2, a^3\}$, then a subgraph is formed K_3. The picture that emerges in the example can be generalized for any $n = p^k$ with p is a prime number and k natural numbers as follows

Theorem 3.3. *Given a dihedral group D_{2n} with $n = p^k$ for a prime number p and a natural number k. The power graph of the dihedral group forms two disjoint subgraphs, where the first subgraph forms a star graph while the other subgraph is a complete graph. S_{n+1} while the other subgraph is a complete graph. K_{n-1}.*

Proof. Based on Theorem 8, the order element of the dihedral group is of the form a^ib for $i = 0, \dots, n-1$ is 2, thus each of these elements is only adjacent to the element e. As a result, the set $\{e, a^ib | i = 0,1,2 \dots, n-1\}$ forms the graph S_{n+1}. Then the element in the dihedral group constructed by a, call it $D = \{a, a^2, \dots, a^{n-1}, e\}$ form a cyclic group with order $n = p^k$. Based on the result of (Chakrabarty et al., 2009), the power graph of D forms a complete graph K_n, then if the vertex e is removed from the set D, then the resulting complete graph becomes K_{n-1}.

Through the result in the Theorem above, we can detail each degree that arises from each vertex formed in the power graph over the dihedral group for $n = p^k$. The theorem related to this is written in (Nurhabibah et al., 2021)We rewrite it with a different proof as follows.

Theorem 3.4. *Given a dihedral group D_{2n} . If $n = p^k$ for a prime number p and a natural number k, then the degree of each vertex in $\Gamma(D_{2n})$ is as follows*

$$der(a^j) = n - 1, der(a^ib) = 1, der(e) = 2n - 1.$$

Proof. It is obvious by looking at the disjoined subgraphs formed, as stated in Theorem 3.3.

As the theorem written above, the power graph over the dihedral group for the $n = p^k$ forms two disjoint subgraphs, where the first subgraph is a star graph and the other subgraph is a complete graph. In general, the vertices in the formed graph can be partitioned into

$$V(\Gamma_{D_{2n}}) = P_0 \cup P_1 \cup P_2$$

Where

$P_0 = \{e\}$

$P_1 = \{a^j | j = 1,2,..n-1\}$

$P_2 = \{a^i b | i = 0,1,\dots,n-1\}$

Then the edges of the formed power graph can be partitioned into

$$E(\Gamma_{Q_{2n}}) = \{[a] \cup [b] \cup [c]\}$$

Where

$[a] = \{ea^j | j = 1,2,3 \dots n-1\}$

$[b] = \{ea^i b | i = 0,2,\dots,n-1\}$

$[c] = \{a^i a^j | i,j = 1,2,\dots,n-1, i \neq j\}$

3.2. Connectivity Index of Power Graph over Dihedral Group for order $n = p^k$

Using the vertex and edge patterns that appear above, we can find the connectivity indices of the power graph of the dihedral group for $n = p^k$ with p is a prime number. The analysis of the connectivity indices starts by looking at the first Zagreb Index involving the degree of each vertex as follows

Theorem 3.5 *Given a power graph Γ over the dihedral group D_{2n}. If $n = p^k$ with p is a prime number, then the First Zagreb index over $\Gamma_{D_{2n}}$ is*

$$M_1(\Gamma_{D_{2n}}) = n^3 + n^2.$$

Proof. Given $n = p^k$ with p is a prime number. The degree of each vertex formed in the power graph has been written in Theorem 3.4, following this result, the first Zagreb Index is as follows

$$\begin{aligned} M_1(\Gamma_{D_{2n}}) &= \sum_{v \in V(\Gamma_{D_{2n}})} (\deg(v))^2 = \deg(e)^2 + \sum_{v \in P_1} \deg(v)^2 + \sum_{v \in P_2} \deg(v)^2 \\ &= (2n-1)^2 + (n-1)(n-1)^2 + n(1)^2 \\ &= 4n^2 - 4n + 1 + n^3 - 3n^2 + 3n - 1 + n \\ &= n^3 + n^2 \end{aligned}$$

Thus, the First Zagreb Index of $\Gamma_{D_{2n}}$ with $n = p^k$ where p is a prime number is $n^3 + n^2$. ■

The next index investigated is the second Zagareb Index whose calculation involves multiplying the degrees of the connected vertices, so the result of Theorem 3.4 is needed again in this calculation.

Theorem 3.6. *Given a power graph Γ over dihedral group Q_{4n}. If $n = p^k$ with p are prime numbers and k a natural number, then the Second Zagreb index over $\Gamma_{Q_{4n}}$ is*

$$M_2(\Gamma_{D_{2n}}) = \frac{n^4 - n^3 + 3n^2 - n}{2}.$$

Proof. Given $n = p^k$ with p is a prime number. Based on Theorem 3.4, we get

$$\begin{aligned} M_2(\Gamma_{D_{2n}}) &= \sum_{uv \in E(\Gamma_{D_{2n}})} \deg(u)\deg(v) \\ &= \sum_{u \in P_1} \deg(e)\deg(u) + \sum_{u \in P_2} \deg(e)\deg(u) + \sum_{uv \in [c]} \deg(u)\deg(v) \\ &= (n-1)(2n-1)(n-1) + n(2n-1) + \left(\binom{n}{2} - (n-1)\right)(n-1)^2 \\ &= (2n^3 - 5n^2 + 4n - 1) + (2n^2 - n) + \frac{(n^4 - 5n^3 + 9n^2 - 7n + 2)}{2} \\ &= \frac{n^4 - n^3 + 3n^2 - n}{2} \end{aligned}$$

So the Second Zagreb Index over $\Gamma_{D_{2n}}$ for $n = p^k$ is $\frac{n^4-n^3+3n^2-n}{2}$. ■

Theorem 3.7. *Given a power graph Γ over dihedral group D_{2n}. The Wiener index of $\Gamma_{Q_{4n}}$ is*

$$W(\Gamma_{D_{2n}}) = \frac{7n^2 - 5n}{2},$$

with $n = p^k$ and p are prime numbers.

Proof. Given $n = p^k$ with p is a prime number, we get

$$\begin{aligned} W(\Gamma_{D_{2n}}) &= \sum_{u,v \in V(\Gamma_{D_{2n}})} d(u,v) \\ &= \sum_{v \in V(\Gamma_{D_{2n}}) - \{e\}} d(e,v) + \sum_{uv \in [c]} d(u,v) + \sum_{u \in P_1, v \in P_2} d(u,v) + \sum_{u,v \in P_2} d(u,v) \\ &= (2n-1) + \left(\binom{n}{2} - (n-1)\right) + n(n-1)(2) + \binom{n}{2}2 \\ &= (2n-1) + \frac{(n-1)(n-2)}{2} + 2(n^2 - n) + n^2 - n = \frac{7n^2 - 5n}{2}. \end{aligned}$$

So the Wiener Index of $\Gamma_{D_{2n}}$ with $n = p^k$ is $\frac{7n^2-5n}{2}$. ■

Theorem 3.8. *Given a power graph Γ over the dihedral group . The hyper-Wiener index over the group is*

$$WW(\Gamma_{D_{2n}}) = 5n^2 - 4n,$$

with $n = p^k$ and p are prime numbers.

Proof. Given $n = p^k$ with p is a prime number. Let us first determine the sum of squared distances between two vertices in $\Gamma_{D_{2n}}$, as follows

$$\sum_{u,v\in V(\Gamma_{D_{2n}})} d(u,v)^2 = \sum_{v\in V(\Gamma_{D_{2n}})-\{e\}} d(e,v)^2 + \sum_{uv\in[c]} d(u,v)^2 + \sum_{u\in P_1,v\in P_2} d(u,v)^2 + \sum_{u,v\in P_2} d(u,v)^2$$
$$= (2n-1)1^2 + \left(\binom{n}{2} - (n-1)\right)1^2 + n(n-1)(2^2) + \binom{n}{2}(2^2)$$
$$= (2n-1) + \frac{(n-1)(n-2)}{2} + 4(n^2-n) + 2(n^2-n) = \frac{13n^2-11n}{2}.$$

Based on Theorem 3.7, we obtain

$$WW(\Gamma_{Q_{4n}}) = \frac{1}{2}\left(W(\Gamma_{Q_{4n}}) + \sum_{u,v\in V(\Gamma_{Q_{4n}})} (d(u,v))^2\right) = \frac{1}{2}\left(\frac{7n^2-5n}{2} + \frac{13n^2-11n}{2}\right)$$
$$= \frac{1}{2}(10n^2-8n) = 5n^2-4n.$$

Thus, the *hyper-Wiener* index over $\Gamma_{D_{2n}}$ with $n = p^k$ where p is a prime number is $5n^2-4n$. ■

Theorem 3.9. *Given a power graph Γ over dihedral group Q_{4n}. The Harary index of $\Gamma_{Q_{4n}}$ is*

$$H(\Gamma_{D_{2n}}) = \frac{5n^2-n}{4},$$

with $n = p^k$ and p are prime numbers.

Proof. It is known that the Harary Index is the sum of the inverse of the distances between two vertices in the $\Gamma_{Q_{4n}}$. Utilizing the result in Theorem 3.7, the Harary Index is obtained as follows

$$H(\Gamma_{D_{2n}}) = \sum_{u,v\in V(\Gamma_{D_{2n}})} \frac{1}{d(u,v)}$$
$$= \sum_{v\in V(\Gamma_{D_{2n}})-\{e\}} \frac{1}{d(e,v)} + \sum_{uv\in[c]} \frac{1}{d(u,v)} + \sum_{u\in P_1,v\in P_2} \frac{1}{d(u,v)} + \sum_{u,v\in P_2} \frac{1}{d(u,v)}$$
$$= (2n-1) + \left(\binom{n}{2} - (n-1)\right) + n(n-1)\left(\frac{1}{2}\right) + \binom{n}{2}\left(\frac{1}{2}\right)$$
$$= (2n-1) + \frac{(n-1)(n-2)}{2} + \frac{n^2-n}{2} + \frac{n(n-1)}{4} = \frac{5n^2-n}{4}.$$

Thus, the Harary Index of $\Gamma_{D_{2n}}$ with $n = p^k$ where p is a prime number is $\frac{5n^2-n}{4}$. ■

Theorem 3.10. *Given a power graph $\Gamma_{D_{2n}}$ over the dihedral group. If $n = p^k$ with p is a prime number, then the Szeged index of $\Gamma_{D_{2n}}$ is*

$$Sz(\Gamma_{D_{2n}}) = 0.$$

Proof. Given $n = p^k$ where p is a prime number. It is observed that

1. Edge $[a] = \{ea^j | j = 1,2,3 \dots n-1\}$
   $N_1(a|\Gamma_{D_{2n}}) = \{u \in V(\Gamma_{D_{2n}}) | d(u,e) < d(u,a^j)\} = \{a^i b\} = P_2,$
   $N_2(a|\Gamma_{D_{2n}}) = \{u \in V(\Gamma_{D_{2n}}) | d(u,a^j) < d(u,e)\} = \emptyset$,
   so that $N_1(a|\Gamma_{D_{2n}}) = n$, and $N_2(a|\Gamma_{D_{2n}}) = 0$.
2. Edge $[b] = \{ea^i b | i = 0,1,2, \dots n-1\}$
   $N_1(b|\Gamma_{D_{2n}}) = \{u \in V(\Gamma_{D_{2n}}) | d(u,e) < d(u,a^i b)\} = P_1 = \{a^j\}$
   $N_2(b|\Gamma_{D_{2n}}) = \{u \in V(\Gamma_{D_{2n}}) | d(u,a^i b) < d(u,e)\} = \emptyset,$

so $N_1(b|\Gamma_{D_{2n}}) = n - 1$ and $N_2(b|\Gamma_{D_{2n}}) = 0$.

3. Edge $[c] = \{a^i a^j | i, j = 1,2, \dots, n-1\}$
   $N_1(c|\Gamma_{D_{2n}}) = \{u \in V(\Gamma_{D_{2n}}) | d(u, a^i) < d(u, a^j)\} = \emptyset$
   $N_2(c|\Gamma_{D_{2n}}) = \{u \in V(\Gamma_{D_{2n}}) | d(u, a^j) < d(u, a^i)\} = \emptyset$,
   so $N_1(c|\Gamma_{D_{2n}}) = 0$ and $N_2(c|\Gamma_{D_{2n}}) = 0$.

Based on the explanation above, the Szeged Index of the graph is as follows

$$Sz(\Gamma_{D_{2n}}) = \sum_{u \in E(\Gamma_{Q_{4n}})} |N_1(u|\Gamma_{D_{2n}})|\,|N_2(u|\Gamma_{D_{2n}})|$$
$$= |N_1(a|\Gamma_{D_{2n}})||N_2(a|\Gamma_{D_{2n}})| + \sum_{b \in [b]} |N_1(b|\Gamma_{D_{2n}})||N_2(b|\Gamma_{D_{2n}})|$$
$$+ \sum_{c \in [c]} |N_1(c|\Gamma_{D_{2n}})||N_2(c|\Gamma_{D_{2n}})| += n.0 + 0\,(n-1) + 0.0 = 0.$$

So the Szeged Index of $\Gamma_{D_{2n}}$ with $n = p^k$ is 0. ■

Overall, the discussion has provided a deeper understanding of how power graphs constructed from dihedral groups can reveal significant structural characteristics when examined through the lens of connectivity indices. In particular, the six indices considered in this study—the First Zagreb, Second Zagreb, Wiener, Hyper-Wiener, Harary, and Szeged—serve as essential tools for interpreting the relationship between group elements and the graphical structures they form. Their combined analysis highlights the rich interaction between algebraic properties and graph invariants, reinforcing the theoretical foundation of this research while also pointing to broader possibilities for interdisciplinary applications.

Conclusion

Six connectivity indices, namely the First Zagreb Index, Second Zagreb Index, Wiener Index, Hyper-Wiener Index, Harary Index, and Szeged Index, can be found on power graphs over the dihedral group for the order of $n = p^k$. These indices are determined by observing the patterns that appear in the graph formed for that order. For future research, these methods can be extended to other classes of groups or to different types of graph invariants to reveal broader structural properties.

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