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Experimental Study of Cable Force Measurement on Cable-Stayed Bridges Based on Vibration Method

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ABSTRACT

This study investigates cable force estimation in cable-stayed bridges through a vibration-based approach, utilizing experimental data measured using an accelerometer sensor. In the initial phase of the research, the frequency data measured by accelerometers is validated through numerical modeling using the Midas Civil software. Additionally, besides employing the string formula, this study adopts formulas proposed by Yu to predict cable forces in two cable-stayed bridges in Indonesia. The estimated cable forces using both formulas are then compared with the actual cable forces measured during the lift-off test. The analysis results indicate that most of the cable frequency data is valid, with differences of less than 7% between the measured frequencies and numerical results. However, a significant difference is observed in one cable, BA-M11, with differences of up to 50.9%. This suggests that the mode order and frequency values measured for this cable are not valid. Through a numerical approach, accurate mode orders and frequencies are determined, enabling confident use of the measurement data for cable force estimation in the case of cable BA-M11. Furthermore, when the validated mode orders and frequency values are used with both the string formula and Yu's proposed formulas, the results show that Yu's formulas tend to provide more accurate estimations compared to the string theory, with average differences in cable force estimation of approximately 4.33% and 2.97% relative to the lift-off force. The contribution of this research lies in the utilization of numerical verification to correct inaccuracies in accelerometer-measured mode orders and frequency values. Subsequently, armed with validated mode orders and frequency values, Yu's proposed formulas demonstrate superior accuracy in predicting cable forces compared to the string theory when both are compared with lift-off test data.

1. Introduction

Cable force evaluation in cable-stayed bridges has undergone extensive research, with various techniques employed. Commonly used methods include hydraulic jack-based lift-off testing [2], [3], permanent devices such as load cells [4], and fiber grating sensors sensitive to temperature and strain changes during bridge construction [5]. In recent years, elasto-magnetic (EM) sensors have garnered attention due to their high sensitivity and rapid response, despite the challenge of calibration [2].

Vibration-based identification methods offer a cost-effective and practical approach for estimating cable forces. This approach, which relies on accelerometer sensors, is used more frequently than static methods due to its simplicity and applicability during both construction and service stages [3]. It involves identifying cable frequencies from ambient vibrations and using them in predetermined formulas or numerical simulations to estimate cable forces.

The ambient vibration method is based on principles akin to string theory, modeling cable vibrations. However, its limitations, particularly for short and long cables, varying boundary conditions, and sag influences, have been demonstrated in further research [6] [7] [8] [9]. Alternatively, beam theory models the cable as a beam under axial loading, providing reliable estimates for short cables but losing effectiveness with sag-influenced cables [10].

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Ongoing research aims to advance vibration-based techniques and address theoretical limitations. A significant challenge in the application of Yu's formula [1] is the requirement for multiple measurable frequencies and mode orders from vibration recordings. However, data obtained from accelerometer sensors may not always provide complete and sequential frequency and mode information. A study indicates that the frequencies of the first mode can be affected by disturbances such as rain, wind, or passing vehicles, which can lead to errors in selecting frequencies and mode orders and compromise the accuracy of cable tension force estimation [11].

This study aims to address these challenges by validating frequency data using numerical models and applying Yu's formula to estimate cable forces on actual bridges in Indonesia. A comparative approach with string theory is also employed. The outcomes of this study are expected to contribute to the development of practical cable force measurement methods for field applications on cable-stayed bridges.

2. Methods

2.1 Object Research

This study was conducted through experimental assessments of two prominent cable-stayed bridges in Indonesia: the RH Fisabilillah Bridge and the Pulau Balang Bridge. The RH Fisabilillah Bridge, connecting Batam Island with Tonton Island in Kepulauan Riau Province, was built in 1998. As illustrated in Figure. 1, it has a total length of 641.8 meters, with the primary span stretching 350 meters. This bridge features a structural design comprising two pylons and 112 support cables.

Moreover, the Pulau Balang Bridge, connecting Balikpapan City with Penajam Paser Utara district in East Kalimantan Province, was built in 2021. Figure. 2 shows the bridge's dimensions, with an overall length of 804 meters and a main span over 402 meters. The Pulau Balang Bridge stands on two pylons and is upheld by a system of 168 cables.

This study selected six cables as representatives of the shortest, intermediate, and longest cables on each bridge. FI-M1, FI-M7, and FI-M14 are the cable codes reviewed on the RH Fisabilillah Bridge, while on the Pulau Balang Bridge, the selected cables are identified as BA-M1, BA-M11, and BA-M21 as shown in Figure. 1 and Figure. 2.

In preparation for the subsequent analysis, it is essential to provide an understanding of the cable properties. The properties, such as the area of the cable cross-section, cable length, and the horizontal angle of the examined cables, are detailed in Table 1. Additionally, the most recent lift-off test data, which represents the actual cable forces, is also included in Table 1.

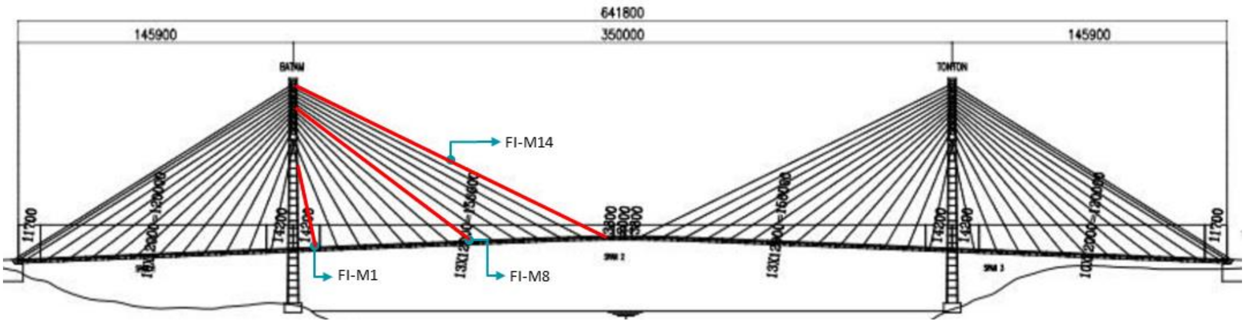

Figure. 1. The location of the examined cables on the RH Fisabilillah Bridge.

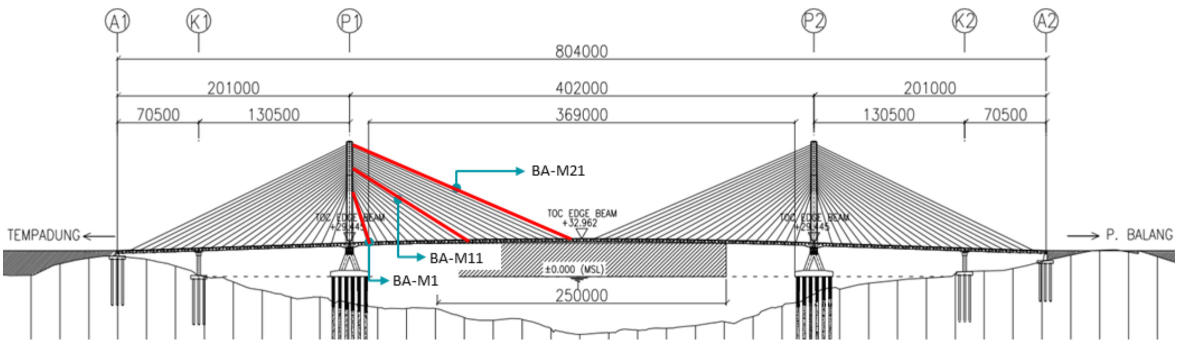

Figure. 2. The location of the examined cables on Pulau Balang Bridge.

Table 1. Cable properties and lift-off test data [12][13]

Cable ID	Area of Strand (m^2)	Cable Length (m)	Horizontal Angle (°)	Lift-off Test (kN)
	A	L	ϴ	T
FI-M1	0.007	60.48	78.55	4419
FI-M7	0.006	110.90	40.76	4320
FI-M14	0.012	187.44	26.33	7693
BA-M1	0.004	50.30	71.51	2887
BA-M11	0.007	124.20	31.89	3928
BA-M21	0.010	212.86	23.31	6602

2.2 Data Collection

The primary data utilized in this research encompasses the ambient vibration signals obtained from experiments, which were measured using a triaxial MEMS accelerometer with a range of ±2g. Figure. 3 illustrates the wave pattern on the cables tensioned at both ends, along with the locations of the vibration sensors. In this experiment, the accelerometer was positioned within the upper limit of the guide pipe with its point at 1/4 of the cable's length (L) in the bottom anchorage. The determination of this position considered the operational range limitations of the manlift as an instrument for sensor installation and to capture frequency data from several modes, especially in specific low modes.

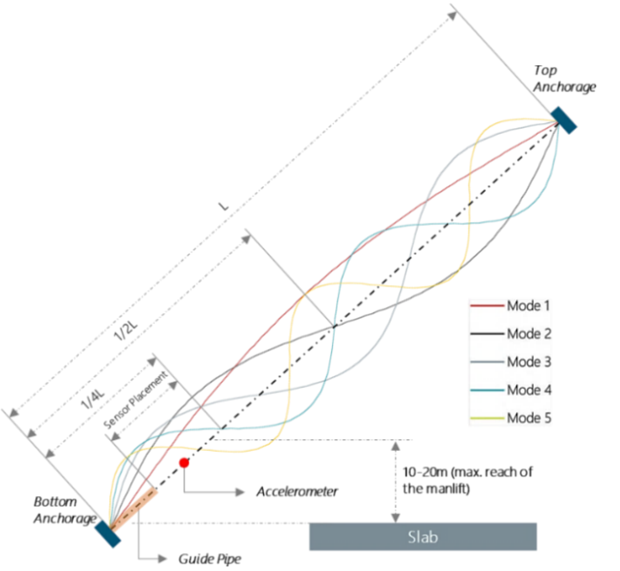

Figure. 3. Schematic of wave distribution along the cable.

The accelerometer is attached to the cable using a bracket and fastening straps, as shown in Figure. 4. Subsequently, ambient vibration recording is conducted with a measurement duration set at 600 seconds and a sampling rate of 256 Hz. The recorded signal is converted from a time domain to a frequency domain using the Fast Fourier Transform (FFT) and a frequency range of 0-14 Hz is selected for remain cables.

Figure. 4. The accelerometer is attached to the cable.

Data collection for ambient vibrations was carried out under the operational conditions of the bridges, during which traffic was present. This meant that the cable vibration recordings using the accelerometer were influenced to some extent by passing vehicles. The recorded vibration signals in the time domain were subsequently transformed into the frequency domain using the Fast Fourier Transform (FFT). Figure. 5 and Figure. 6 depict the time domain and frequency domain graphs for cables FI-M1 and FI-M7, respectively. Notably, when examining the number of recorded peaks in the frequency domain, there is a significant difference between cables FI-M1 and FI-M7. This difference suggests that cable length has a substantial impact on the number of modes captured. Longer cables tend to capture a greater number of total modes, and vice versa.

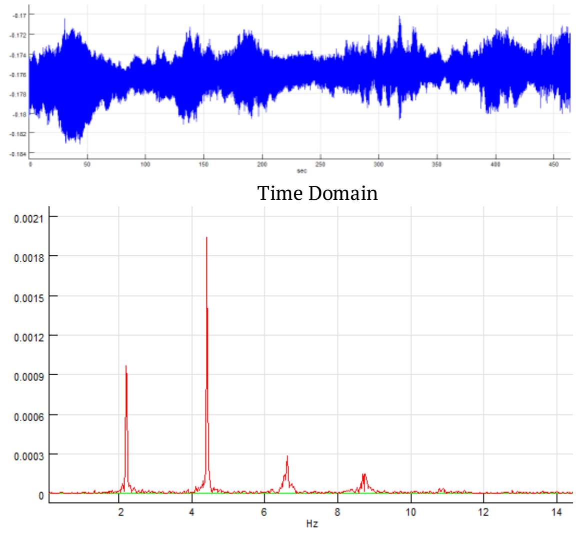

Figure. 5. The recorded vibration for cable FI-M1.

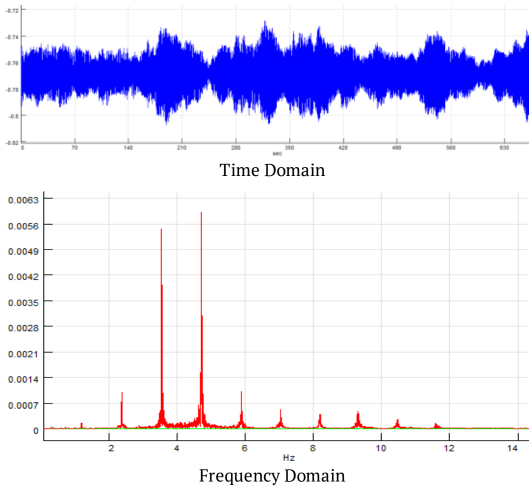

Figure. 6. The recorded vibration for cable FI-M7.

However, it's important to note that the FFT results from the vibration recordings may not always present the exact mode order and frequency values. Validation of the measured frequency and mode order data is necessary to ensure accurate estimation of cable forces, especially given that vibration recordings were conducted during the operational phase of the bridges, making them subject to the influence of passing vehicles.

Therefore, to confirm the accuracy of the frequency data obtained from the FFT process, initial assumptions are needed. The assumed frequency and mode order data obtained from field measurements can be seen in Table 2. These values from the accelerometer require validation, which will be addressed in the next subsection using a numerical approach.

Table 2. Measured frequencies (pre assumes)

Cable ID	Measured Natural Frequency (Hz)				
	f1	f2	f3	f4	f5
FI-M1	2.19	4.4	6.6	8.76	-
FI-M7	1.23	2.47	3.62	4.82	6.06
FI-M14	0.72	1.44	2.14	2.88	3.52
BA-M1	2.78	5.34	-	-	-
BA-M11	2.12	3.15	4.16	5.18	-
BA-M21	0.63	1.25	1.87	2.48	3.1

2.3 Validation of The Measured Frequency

The frequency data and mode order obtained from the accelerometer were validated using a numerical model using the Midas Civil software. The cable is modeled as a simple string using one-dimensional (1D) elements with dimensions adjusted to the cable properties outlined in Table 1. Then, these elements are provided with hinged boundaries at both ends. Additionally, the cable was discretized into smaller elements to ensure the model accurately captured the required mode shapes. The numerical model representation of the cable is illustrated in Figure. 7, where "L" denotes the total length of the cable and "θ" signifies the angle formed by the cable with respect to its horizontal direction.

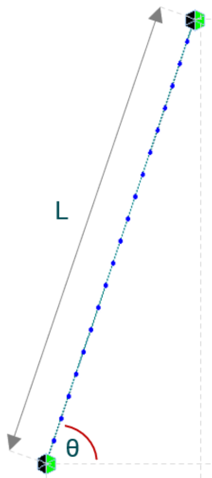

Figure. 7. Numerical model of the inclined cable.

The cable forces are modeled as axial tension forces acting on each divided element. This study applies these axial forces using lift-off data to represent the actual cable forces. The axial forces are initially modeled as initial forces to calculate the geometric stiffness in a general linear analysis [14]. In the subsequent step of numerical modeling, an eigenvalue and eigenvector analysis characterize the vibration attributes of the structural system. Eigenvalues represented the system's resonant frequencies, while eigenvectors illustrated the shapes or vibration patterns associated with each resonance mode [15].

Following the numerical modeling process, the frequency values and mode orders obtained from the model are compared with the measurement data. The assessment results of this comparison serve as validation parameters for the measured frequencies, providing a benchmark for accuracy. In cases of significant discrepancies, necessary adjustments to the frequency data and mode orders are made. However, if the comparative analysis indicates a low percentage of error, the measurement data can be deemed valid and subsequently used for calculating cable force estimates.

Mode numbers along with their corresponding frequency values serve as crucial parameters for calculating cable force estimates. Consequently, validating measurement data obtained from accelerometers with the numerical approaches becomes imperative in this study.

2.4 Cable Force Estimation

After validating the frequency data obtained from ambient vibration using the numerical method proposed in the previous subsection, cable force estimation is performed using a widely used formula known as the string theory by [16]. The cable force calculation in the string theory formula is described in Equation (1) below:

$$T = 4mL^2\left(\frac{f_n}{n}\right)^2 \quad (1)$$

Where T represents the cable force estimate, m is the mass per unit length, L is the cable length, fn is the frequency, and n indicates the mode order.

The string theory formula requires only one set of measured frequency and mode order. The most reliable mode used in calculating cable force is usually the first mode [1]. Therefore, when using this formula, first mode data must be available and reliable. Otherwise, the mentioned calculation would face difficulties.

To overcome this limitation, Yu proposes an alternative formulation that eliminates the use of the first mode frequency and flexural stiffness [1]. This provides an alternative way for researchers to verify cable force estimates. This formula employs an analytical beam formulation with simple supports to illustrate the idea of calculating cable force using only two arbitrary mode frequencies, as shown in Equations (2) and (2a) below:

$$T = 4(\rho A)L^2\bar{f_c}^2 \quad (2)$$

with

$$\bar{f_c}^2 = \left[\frac{p^4{f_n}^2 - n^4{f_p}^2}{n^2p^2(n^2-p^2)}\right] \quad (2a)$$

Where T represents the cable force estimate, ρ stands for the density of steel, A denotes the cross-sectional area of the strand, L is the cable length, p signifies the previous mode, fp is the previous frequency, n indicates the latter mode, and fn represents the latter frequency.

Yu's proposed formula is that it allows for using two pairs of modes and frequencies that do not necessarily have to be sequential, making it applicable to data sets with incomplete mode numbers. In this study, the value of $\bar{f_c}^2$ is calculated using all pairs of frequencies from adjacent mode numbers and pairs of frequencies with distant mode numbers. Subsequently, the cable force estimation (T) calculated using these frequency pairs is averaged to produce the final estimation value.

Cable force estimates were calculated using two different formulas: the string theory formula proposed by Irvine [16] and the formula introduced by Yu. The outcomes of these estimations, whether using the string formula or Yu's formula, are compared to the actual cable force. The actual cable forces employed in this research were derived from the lift-off force data listed in Table 1. The difference between the estimated cable force and the actual force is then presented as a percentage, representing the accuracy of using the formula.

3. Result and Discussion

In this study, the initial step following the measurement of frequencies on the actual bridge is to validate the collected data. The proposed solution involves numerical modeling of the cable, as previously described. The validation process yields the percentage difference between the numerical frequencies and the measurements for all the cables under investigation, as listed in Table 3.

The comparative results between the measurement and numerical frequencies in Table 3 reveal that for cables FI-M1, FI-M7, FI-M14, BA-M1, and BA-M21, the differences are less than 7%. These insignificant differences indicate that the mode order and frequency values from the measurements are valid and ready to be used for cable force estimation.

However, a highly significant difference is observed for cable BA-M11, with variations ranging from 19.7% to 50.9%. These findings indicate inaccuracies in the initial assumptions regarding the mode order and measured frequency values for this cable. According to Table 3, in the first mode of BA-M11, the numerical frequency is 1.04Hz, whereas the first frequency observed in the measurements is 2.12Hz. This discrepancy highlights errors in the initial assumptions, necessitating adjustments to the mode number and frequency values based on the numerical model. By examining the second mode of the numerical model, which is 2.08Hz, the measured frequency of 2.12Hz aligns with the second mode. Additionally, to validate the subsequent mode order and frequency values, the same approach can be applied. Consequently, the validation results can be presented, as shown in Table 4.

Table 3. Comparison between the measured and the numerical model frequencies

Mode	Frequency (Hz)		Error (%)
	Measured	Numerical Model	
FI-M1			
1	2.19	2.20	0.3
2	4.4	4.38	0.4
3	6.6	6.55	0.7
4	8.76	8.69	0.8
5	-	10.80	-
FI-M7			
1	1.23	1.25	1.8
2	2.47	2.51	1.4
3	3.62	3.75	3.6
4	4.82	5.00	3.6
5	6.06	6.24	2.9
FI-M14			
1	0.72	0.74	2.3
2	1.44	1.47	2.3
3	2.14	2.21	3.1
4	2.88	2.94	2.2
5	3.52	3.68	4.3

Mode	Frequency (Hz)		Error (%)
	Measured	Numerical Model	
BA-M1			
1	2.78	2.85	2.5
2	5.34	5.69	6.1
3	-	8.49	-
4	-	11.23	-
5	-	13.91	-
BA-M11			
1	2.12	1.04	50.9%
2	3.15	2.08	34.0%
3	4.16	3.12	25.0%
4	5.18	4.16	19.7%
5	-	5.18	-
BA-M21			
1	0.63	0.64	1.3
2	1.25	1.28	2.1
3	1.87	1.91	2.3
4	2.48	2.55	2.8
5	3.1	3.19	2.7

Table 4. The validated mode order and frequency for cable BA-M11

Mode	Frequency (Hz)		Error (%)
	Measured	Numerical Model	
BA-M11			
1	-	1.04	-
2	2.12	2.08	1.9%
3	3.15	3.12	1.0%
4	4.16	4.16	0.0%
5	5.18	5.18	0.0%

Table 4 demonstrates the validity of the second mode and frequency values obtained from measurement data. This is evident from the insignificant percentage differences between the measurement and numerical data. The largest difference is observed in the second mode, which amounts to 1.9%, while the fourth and fifth modes show identical values with a 0.0% difference. Based on these validation results, the measurement frequencies for cable BA-M11 can be confidently used for cable force estimation.

The next stage of this study involves calculating the estimated cable forces using Equations (1) and (2), which will subsequently be compared to the lift-off forces of each cable. The analysis results for all six cables are presented in Table 5.

According to the cable force calculations presented in Table 5, when applying the string theory for cable force calculations, the smallest deviation is 0.29% from the lift-off force for cable FI-M1, while the most significant deviation is 8.84% from the lift-off force for cable BA-M1. On the other hand, the formula proposed by [1] exhibits a minor deviation, measuring just 0.15% from the lift-off force for cable FI-M1. Conversely, the most significant is observed in the case of cable FI-M7, reaching as high as 4.95% from the lift-off force.

The difference between the lift-off force and the string formula averages 4.33%, while it is 2.97% when using the Yu formula. Consequently, based on these findings, the cable force estimation formula proposed by Yu is considered more accurate in predicting cable force than the string formula when both are compared with the lift-off force.

. **Table 5.** Calculation of cable force

Cable ID	Cable Length (m)	Lift Off Test (kN)	String Theory, Irvine (1981)		Yu (2011)	
			Force Est. (kN)	T vs $T_{est.1}$	Force Est. (kN)	T vs $T_{est.2}$
	L	T	$T_{est.1}$	Δ_1	$T_{est.2}$	Δ_2
FI-M1	60.48	4419	4406.09	0.29%	4425.53	0.15%
FI-M7	110.90	4320	4078.57	5.59%	4106.35	4.95%
FI-M14	187.44	7693	7249.20	5.77%	7433.93	3.37%
BA-M1	50.30	2887	2631.93	8.84%	2808.93	2.71%
BA-M11	124.20	3928	3966.26	0.97%	4047.31	2.96%
BA-M21	212.86	6602	6303.31	4.53%	6358.64	3.69%

4. Conclusion

This study investigates a vibration-based approach for estimating cable tension forces in cable-stayed bridges using experimental data. The validation of measured frequency data via numerical modeling demonstrates favorable results for most examined cables, except for one that exhibited a significant discrepancy between measured frequency data and numerical results, primarily due to mode order placement issues. Through numerical analysis, correct mode orders and frequencies were determined, allowing for confident use of the measured frequency data for cable force estimation with available formulas. Notably, Yu's formula outperforms the string theory in predicting cable forces, with differences in estimation results averaging 4.33% for the string formula and 2.97% for Yu's formula when compared to the lift-off force. Additionally, the use of numerical modeling to validate measured frequency data enhances the accuracy of cable force estimation, contributing to practical cable force measurement methods in cable-stayed bridges. Nevertheless, some limitations, including the number of examined cables and unaccounted-for boundary conditions, may impact cable force estimation results, suggesting opportunities for further research to address these issues and develop more practical and precise cable force identification methods.

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Disclaimer

The authors state that there are no conflicts of interest regarding the publication of this paper.

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